

Základní goniometrické rovnice (4)

1. Sinus

1) Řešte v R rovnice:

$$\sin x = \frac{\sqrt{2}}{2}$$

$$\text{VH: } x_1 = \frac{\pi}{4} + 2k\pi, x_2 = \frac{3\pi}{4} + 2k\pi, k \in Z$$

2) Řešte v R rovnice:

$$\sin x = -\frac{\sqrt{2}}{2}$$

$$\text{VH: } x_1 = \frac{5\pi}{4} + 2k\pi, x_2 = \frac{7\pi}{4} + 2k\pi, k \in Z$$

3) Řešte v R rovnice:

$$\sin x = -\frac{\sqrt{3}}{2}$$

$$\text{VH: } x_1 = \frac{4\pi}{3} + 2k\pi, x_2 = \frac{5\pi}{3} + 2k\pi, k \in Z$$

4) Řešte v R rovnice:

$$\sin x = \frac{\sqrt{3}}{2}$$

$$\text{VH: } x_1 = \frac{\pi}{3} + 2k\pi, x_2 = \frac{2\pi}{3} + 2k\pi, k \in Z$$

5) Řešte v R rovnice:

$$\sin x = \frac{1}{2}$$

$$\text{VH: } x_1 = \frac{\pi}{6} + 2k\pi, x_2 = \frac{5\pi}{6} + 2k\pi, k \in Z$$

6) Řešte v R rovnice:

$$\sin x = -\frac{1}{2}$$

$$\text{VH: } x_1 = \frac{7\pi}{6} + 2k\pi, x_2 = \frac{11\pi}{6} + 2k\pi, k \in Z$$

7) Řešte v R rovnice:

$$\sin x = 1$$

$$\text{VH: } x = \frac{\pi}{2} + 2k\pi, k \in Z$$

8) Řešte v R rovnice:

$$\sin x = -1$$

$$\text{VH: } x = \frac{3\pi}{2} + 2k\pi, k \in Z$$

9) Řešte v R rovnice:

$$\sin x = 0$$

$$\text{VH: } x = 0 + k\pi, k \in Z$$

10) Řešte v R rovnice:

$$\sin x = \sqrt{3}$$

$$\text{VH: } NR$$

11) Řešte v R rovnice:

$$\sin x = -2$$

$$\text{VH: } NR$$

12) Řešte v R rovnice:

$$\sin x = 1,1$$

$$\text{VH: } NR$$

2. Cosinus

1) Řešte v R rovnice:

$$\cos x = \frac{\sqrt{2}}{2}$$

$$\text{VH: } x_1 = \frac{\pi}{4} + 2k\pi, x_2 = \frac{7\pi}{4} + 2k\pi, k \in Z$$

2) Řešte v R rovnice:

$$\cos x = -\frac{\sqrt{2}}{2}$$

$$\text{VH: } x_1 = \frac{3\pi}{4} + 2k\pi, x_2 = \frac{5\pi}{4} + 2k\pi, k \in Z$$

3) Řešte v R rovnice:

$$\cos x = -\frac{\sqrt{3}}{2}$$

$$\text{VH: } x_1 = \frac{5\pi}{6} + 2k\pi, x_2 = \frac{7\pi}{6} + 2k\pi, k \in Z$$

4) Řešte v R rovnice:

$$\cos x = \frac{\sqrt{3}}{2}$$

$$\text{VH: } x_1 = \frac{\pi}{6} + 2k\pi, x_2 = \frac{11\pi}{6} + 2k\pi, k \in Z$$

5) Řešte v R rovnice:

$$\cos x = \frac{1}{2}$$

$$\text{VH: } x_1 = \frac{\pi}{3} + 2k\pi, x_2 = \frac{5\pi}{3} + 2k\pi, k \in Z$$

6) Řešte v R rovnice:

$$\cos x = -\frac{1}{2}$$

$$\text{VH: } x_1 = \frac{2\pi}{3} + 2k\pi, x_2 = \frac{4\pi}{3} + 2k\pi, k \in Z$$

7) Řešte v R rovnice:

$$\cos x = 1$$

$$\text{VH: } x = 2k\pi, k \in Z$$

8) Řešte v R rovnice:

$$\cos x = -1$$

$$\text{VH: } x = \pi + 2k\pi, k \in Z$$

9) Řešte v R rovnice:

$$\cos x = 0$$

$$\text{VH: } x = \frac{\pi}{2} + k\pi, k \in Z$$

10) Řešte v R rovnice:

$$\cos x = -\frac{3}{2}$$

$$\text{VH: } NR$$

11) Řešte v R rovnice:

$$\cos x = -\sqrt{3}$$

$$\text{VH: } NR$$

12) Řešte v R rovnice:

$$\cos x = 3$$

$$\text{VH: } NR$$

3. Tangens

1) Řešte v R rovnice:

$$\text{tg } x = \frac{1}{\sqrt{3}}$$

$$\text{VH: } x = \frac{\pi}{6} + k\pi, k \in Z$$

2) Řešte v R rovnice:

$$\text{tg } x = -\frac{1}{\sqrt{3}}$$

$$\text{VH: } x = \frac{5\pi}{6} + k\pi, k \in Z$$

3) Řešte v R rovnice:

$$\text{tg } x = \sqrt{3}$$

$$\text{VH: } x = \frac{\pi}{3} + k\pi, k \in Z$$

4) Řešte v R rovnice:

$$\text{tg } x = -\sqrt{3}$$

$$\text{VH: } x = \frac{2\pi}{3} + k\pi, k \in Z$$

5) Řešte v R rovnice:

$$\text{tg } x = 1$$

$$\text{VH: } x = \frac{\pi}{4} + k\pi, k \in Z$$

6) Řešte v R rovnice:

$$\text{tg } x = -1$$

$$\text{VH: } x = \frac{3\pi}{4} + k\pi, k \in Z$$

7) Řešte v R rovnice:

$$\text{tg } x = 0$$

$$\text{VH: } x = k\pi, k \in Z$$

8) Řešte v R rovnice:

$$\text{tg } x = NR$$

$$\text{VH: } x = \frac{\pi}{2} + k\pi, k \in Z$$

4. Cotangens

1) Řešte v R rovnice:

$$\text{cotg } x = \frac{1}{\sqrt{3}}$$

$$\text{VH: } x = \frac{\pi}{3} + k\pi, k \in Z$$

2) Řešte v R rovnice:

$$\text{cotg } x = -\frac{1}{\sqrt{3}}$$

$$\text{VH: } x = \frac{2\pi}{3} + k\pi, k \in Z$$

3) Řešte v R rovnice:

$$\text{cotg } x = \sqrt{3}$$

$$\text{VH: } x = \frac{\pi}{6} + k\pi, k \in Z$$

4) Řešte v R rovnice:

$$\text{cotg } x = -\sqrt{3}$$

$$\text{VH: } x = \frac{5\pi}{6} + k\pi, k \in Z$$

5) Řešte v R rovnice:

$$\text{cotg } x = 1$$

$$\text{VH: } x = \frac{\pi}{4} + k\pi, k \in Z$$

6) Řešte v R rovnice:

$$\text{cotg } x = -1$$

$$\text{VH: } x = \frac{3\pi}{4} + k\pi, k \in Z$$

7) Řešte v R rovnice:

$$\text{cotg } x = 0$$

$$\text{VH: } x = \frac{\pi}{2} + k\pi, k \in Z$$

8) Řešte v R rovnice:

$$\text{cotg } x = NR$$

$$\text{VH: } x = k\pi, k \in Z$$

5. Substituce – sin (rad)

1) Řešte v R rovnici:

$$\sin(x - \frac{\pi}{4}) = 0$$

$$\text{Sb-MM: } x_1 = \frac{1}{4}\pi + k\pi \dots \text{str.76/3.9-d}$$

2) Řešte v R rovnici:

$$2 \sin \frac{1}{2} x = \sqrt{3}$$

$$\text{Sb-MM:}$$

$$x_1 = \pi + 6k\pi, x_2 = 2\pi + 6k\pi \dots \text{str.76/3.9-c)}$$

3) Řešte v R rovnici:

$$\sin(2x + \frac{\pi}{6}) = -1$$

$$\text{Sb-rce } x_1 = \frac{2}{3}\pi + k\pi \dots \text{str.173/5.3-1)}$$

4) Řešte v R rovnici:

$$\sin(2x + \frac{\pi}{3}) = -\frac{\sqrt{3}}{2}$$

$$\text{Sb-rce}$$

$$x_1 = \frac{1}{2}\pi + k\pi, x_2 = \frac{2}{3}\pi + k\pi \dots \text{str.173/5.3-2)}$$

5) Řešte v R rovnici:

$$\sin(4x - \frac{\pi}{2}) = \frac{1}{2}$$

$$\text{VH: } x_1 = \frac{1}{8}\pi + \frac{k\pi}{2}, x_2 = \frac{7}{24}\pi + \frac{k\pi}{2}$$

6) Řešte v R rovnici:

$$\sin(4x - \frac{\pi}{3}) = -\frac{1}{2}$$

$$\text{VH: } x_1 = \frac{3}{8}\pi + \frac{k\pi}{2}, x_2 = \frac{13}{24}\pi + \frac{k\pi}{2}$$

6. Substituce – cos (rad)

1) Řešte v R rovnici:

$$\cos(2x - \frac{\pi}{2}) = 1$$

$$\text{Sb-MM: } x_1 = \frac{1}{4}\pi + k\pi \dots \text{str.76/3.9-e)}$$

2) Řešte v R rovnici:

$$\cos(x - \frac{\pi}{4}) = \frac{\sqrt{2}}{2}$$

$$\text{Sb-rce:}$$

$$x_1 = \frac{1}{2}\pi + 2k\pi, x_2 = 2k\pi \dots \text{str.174/5.3-3)}$$

3) Řešte v R rovnici:

$$\cos(2x - \frac{\pi}{4}) = -1$$

Sb-rce $x_1 = \frac{5}{8}\pi + k\pi \dots$ str.174/5.3-4)

4) Řešte v R rovnici:

$$\frac{1}{\sqrt{5}} \cos(2x + \frac{\pi}{3}) = -\frac{\sqrt{5}}{5}$$

Sb-rce $x_1 = \frac{1}{3}\pi + k\pi \dots$ str.174/5.3-6)

5) Řešte v R rovnici:

$$\cos(4x - \frac{\pi}{2}) = \frac{1}{2}$$

$$\text{VH: } x_1 = \frac{1}{6}\pi + \frac{k\pi}{2}, x_2 = \frac{1}{2}\pi + \frac{k\pi}{2}$$

6) Řešte v R rovnici:

$$\cos(4x - \frac{\pi}{3}) = -\frac{1}{2}$$

$$\text{VH: } x_1 = \frac{1}{4}\pi + \frac{k\pi}{2}, x_2 = \frac{5}{12}\pi + \frac{k\pi}{2}$$

7. Substitute – sin, cos (°)

1) Řešte v R rovnici:

$$\sin(2x - 20^\circ) = \frac{1}{2}$$

$$\text{VH: } x_1 = 25^\circ + k \cdot 180^\circ, x_2 = 85^\circ + k \cdot 180^\circ$$

2) Řešte v R rovnici:

$$\sin(2x - 20^\circ) = -\frac{1}{2}$$

VH:

$$x_1 = 115^\circ + k \cdot 180^\circ, x_2 = 175^\circ + k \cdot 180^\circ$$

3) Řešte v R rovnici:

$$\sin(2x - 10^\circ) = -\frac{\sqrt{3}}{2}$$

VH:

$$x_1 = 125^\circ + k \cdot 180^\circ, x_2 = 155^\circ + k \cdot 180^\circ$$

4) Řešte v R rovnici:

$$\sin(2x - 10^\circ) = \frac{\sqrt{3}}{2}$$

$$\text{VH: } x_1 = 35^\circ + k \cdot 180^\circ, x_2 = 65^\circ + k \cdot 180^\circ$$

5) Řešte v R rovnici:

$$\cos(3x + 90^\circ) = \frac{\sqrt{2}}{2}$$

VH:

$$x_1 = -15^\circ + k \cdot 120^\circ, x_2 = 75^\circ + k \cdot 120^\circ$$

6) Řešte v R rovnici:

$$\cos(3x + 90^\circ) = -\frac{\sqrt{2}}{2}$$

$$\text{VH: } x_1 = 15^\circ + k \cdot 120^\circ, x_2 = 45^\circ + k \cdot 120^\circ$$

7) Řešte v R rovnici:

$$\cos(3x + 180^\circ) = \frac{\sqrt{2}}{2}$$

VH:

$$x_1 = -45^\circ + k \cdot 120^\circ, x_2 = 45^\circ + k \cdot 120^\circ$$

8) Řešte v R rovnici:

$$\cos(3x + 180^\circ) = -\frac{\sqrt{2}}{2}$$

VH:

$$x_1 = -15^\circ + k \cdot 120^\circ, x_2 = 15^\circ + k \cdot 120^\circ$$

8. Substitute – tg (rad)

1) Řešte v R rovnici:

$$\frac{2}{\sqrt{3}} \operatorname{tg}(\frac{x}{2} + \frac{\pi}{4}) = -\frac{2\sqrt{3}}{3}$$

Sb-rce: $x_1 = \pi + 2k\pi \dots$ str.174/5.3-7)

2) Řešte v R rovnici:

$$\operatorname{tg}(\frac{3}{4}x - \frac{1}{3}\pi) = -\sqrt{3}$$

Sb-MM: $x_1 = \frac{4k\pi}{3} \dots$ str.76/3.9-f)

3) Řešte v R rovnici:

$$\operatorname{tg}(2x - \frac{\pi}{2}) = 1$$

$$\text{VH: } x_1 = \frac{3}{8}\pi + \frac{k\pi}{2}$$

4) Řešte v R rovnici:

$$\operatorname{tg}(2x - \frac{\pi}{2}) = -1$$

$$\text{VH: } x_1 = \frac{5}{8}\pi + \frac{k\pi}{2}$$

9. Substitute – cotg (rad)

1) Řešte v R rovnici:

$$\cot g(2x - \frac{\pi}{2}) = 1$$

$$\text{VH: } x_1 = \frac{3}{8}\pi + \frac{k\pi}{2}$$

2) Řešte v R rovnici:

$$\cot g(2x - \frac{\pi}{2}) = -1$$

$$\text{VH: } x_1 = \frac{5}{8}\pi + \frac{k\pi}{2}$$

3) Řešte v R rovnici:

$$\cot g(\frac{x}{2} - \frac{\pi}{3}) = -\sqrt{3}$$

Sb-rce: $x_1 = \frac{2}{3}\pi + 2k\pi \dots$ str.174/5.3-6)

4) Řešte v R rovnici:

$$\cot g(3x) = -\frac{1}{\sqrt{3}}$$

$$\text{VH: } x_1 = \frac{2}{9}\pi + \frac{k\pi}{3} = 40^\circ + k \cdot 60^\circ$$

10. Substitute – tg, cotg (°)

1) Řešte v R rovnici:

$$\operatorname{tg}(\frac{1}{2}x - 30^\circ) = 0$$

$$\text{VH: } x_1 = 60^\circ + k \cdot 360^\circ$$

2) Řešte v R rovnici:

$$\operatorname{tg}(\frac{1}{2}x - 30^\circ) = N\check{R}$$

$$\text{VH: } x_1 = 240^\circ + k \cdot 360^\circ$$

3) Řešte v R rovnici:

$$\operatorname{tg}(\frac{1}{2}x - 30^\circ) = \sqrt{3}$$

$$\text{VH: } x_1 = 180^\circ + k \cdot 360^\circ$$

4) Řešte v R rovnici:

$$\operatorname{tg}(\frac{1}{2}x - 30^\circ) = \frac{1}{\sqrt{3}}$$

$$\text{VH: } x_1 = 120^\circ + k \cdot 360^\circ$$

5) Řešte v R rovnici:

$$\operatorname{tg}(3x - 90^\circ) = 1$$

$$\text{VH: } x_1 = 45^\circ + k \cdot 60^\circ$$

6) Řešte v R rovnici:

$$\operatorname{tg}(3x - 90^\circ) = -1$$

$$\text{VH: } x_1 = 75^\circ + k \cdot 60^\circ$$

7) Řešte v R rovnici:

$$\cot g(2x) = 0$$

$$\text{VH: } x_1 = 45^\circ + k \cdot 90^\circ$$

8) Řešte v R rovnici:

$$\cot g(2x) = N\check{R}$$

$$\text{VH: } x_1 = 0^\circ + k \cdot 90^\circ$$

9) Řešte v R rovnici:

$$\cot g(2x) = -\sqrt{3}$$

$$\text{VH: } x_1 = 75^\circ + k \cdot 90^\circ$$

10) Řešte v R rovnici:

$$\cot g(2x) = -\frac{1}{\sqrt{3}}$$

$$\text{VH: } x_1 = 60^\circ + k \cdot 90^\circ$$

11) Řešte v R rovnici:

$$\cot g(3x - \frac{\pi}{2}) = 1$$

$$\text{VH: } x_1 = 45^\circ + k \cdot 60^\circ$$

12) Řešte v R rovnici:

$$\cot g(3x - 90^\circ) = -1$$

$$\text{VH: } x_1 = 75^\circ + k \cdot 60^\circ$$

11. Rovnice v intervalu

1) Řešte v intervalu $\langle 0; 2\pi \rangle$:

$$\sin x = -\frac{\sqrt{3}}{2}$$

$$\text{VH: } x_1 = \frac{4\pi}{3}, x_2 = \frac{5\pi}{3}$$

2) Řešte v intervalu $\langle -\pi; 2\pi \rangle$:

$$\sin x = \frac{1}{2}$$

$$\text{VH: } x_1 = \frac{\pi}{6}, x_2 = \frac{5\pi}{6}$$

3) Řešte v intervalu $\langle 0; \pi \rangle$:

$$\sin x = -1$$

$$\text{VH: } N\check{R}$$

4) Řešte v intervalu $\langle 0; 2\pi \rangle$:

$$\sin x = 0$$

$$\text{VH: } x_1 = 0, x_2 = \pi, x_3 = 2\pi$$

5) Řešte v intervalu $\langle 0; 2\pi \rangle$:

$$\cos x = 1$$

$$\text{VH: } x = 2\pi$$

6) Řešte v intervalu $\langle -2\pi; 2\pi \rangle$:

$$\cos x = \frac{\sqrt{2}}{2}$$

$$\text{VH: } x_1 = -\frac{7\pi}{4}, x_2 = -\frac{\pi}{4}, x_3 = \frac{\pi}{4}, x_4 = \frac{7\pi}{4}$$

7) Řešte v intervalu $\langle 0; \pi \rangle$:

$$\cos x = 0$$

$$\text{VH: } x = \frac{\pi}{2}$$

8) Řešte v intervalu $\langle 0; 2\pi \rangle$:

$$\cos x = -\frac{3}{2}$$

$$\text{VH: } N\check{R}$$

9) Řešte v intervalu $\langle 0; 2\pi \rangle$:

$$\operatorname{tg} x = \frac{1}{\sqrt{3}}$$

$$\text{VH: } x_1 = \frac{\pi}{6}, x_2 = \frac{7\pi}{6}$$

10) Řešte v intervalu $\langle 0; \pi \rangle$:

$$\operatorname{tg} x = 1$$

$$\text{VH: } x = \frac{\pi}{4}$$

11) Řešte v intervalu $\langle 0; 2\pi \rangle$:

$$\operatorname{tg} x = 0$$

$$\text{VH: } x = \pi$$

12) Řešte v intervalu $\langle \pi; 2\pi \rangle$:

$$\operatorname{tg} x = -\sqrt{3}$$

$$\text{VH: } x = \frac{5\pi}{3}$$

13) Řešte v intervalu $\langle -\pi; 0 \rangle$:

$$\cot g x = \sqrt{3}$$

$$\text{VH: } x = -\frac{5\pi}{6}$$

14) Řešte v intervalu $\langle 0; 2\pi \rangle$:

$$\cot g x = 0$$

$$\text{VH: } x_1 = \frac{\pi}{2}, x_2 = \frac{3\pi}{2}$$

15) Řešte v intervalu $\langle 0; 2\pi \rangle$:

$$\cot g x = N\check{R}$$

$$\text{VH: } x_1 = 0, x_2 = \pi, x_3 = 2\pi$$

16) Řešte v intervalu $\langle \pi; 2\pi \rangle$:

$$\cot g x = -1$$

$$\text{VH: } x = \frac{7\pi}{4}$$