

Goniometrické rovnice (4)

1. Základní úprava

1) Řešte v **R**:

$$\frac{5 + \sin x}{1 - \sin x} = 3$$

Sb-MM: $y = -\frac{1}{2}$; $x_1 = \frac{7}{6}\pi + 2k\pi$, $x_2 = \frac{11}{6}\pi + 2k\pi \dots \text{str.77/5.1-b)}$

2) Řešte v **R**:

$$\frac{5 + \cos x}{1 - \cos x} = 3$$

VH: $y = -\frac{1}{2}$; $x_1 = \frac{2}{3}\pi + 2k\pi$, $x_2 = \frac{4}{3}\pi + 2k\pi$

3) Řešte v **R**:

$$\frac{4 + \operatorname{tg} x}{2 - \operatorname{tg} x} = 1$$

VH: $y = -1$; $x = \frac{3}{4}\pi + k\pi$

4) Řešte v **R**:

$$\frac{3 + \cot gx}{3 - \cot gx} = 1$$

VH: $y = 0$; $x = \frac{1}{2}\pi + k\pi$

2. Plus mínus odmocnina

1) Řešte v **R**:

$$3\operatorname{tg}^2 x = 1$$

Sb-MM: $(y + \frac{1}{\sqrt{3}})(y - \frac{1}{\sqrt{3}}) = 0$; $x_1 = \frac{5}{6}\pi + k\pi$, $x_2 = \frac{1}{6}\pi + k\pi \dots \text{str.77/5.1-d)}$

2) Řešte v **R**:

$$\operatorname{tg}^2 x = 3$$

VH: $(y + \sqrt{3})(y - \sqrt{3}) = 0$; $x_1 = \frac{1}{3}\pi + k\pi$, $x_2 = \frac{2}{3}\pi + k\pi$

3) Řešte v **R**:

$$3\cot g^2 x = 1$$

VH: $(y + \frac{1}{\sqrt{3}})(y - \frac{1}{\sqrt{3}}) = 0$; $x_1 = \frac{1}{3}\pi + k\pi$, $x_2 = \frac{2}{3}\pi + k\pi$

4) Řešte v **R**:

$$\cot g^2 x = 3$$

VH: $(y + \sqrt{3})(y - \sqrt{3}) = 0$; $x_1 = \frac{1}{6}\pi + k\pi$, $x_2 = \frac{5}{6}\pi + k\pi$

5) Řešte v **R**:

$$\cos^2 x = \frac{3}{4}$$

VH: $(y + \frac{\sqrt{3}}{2})(y - \frac{\sqrt{3}}{2}) = 0$; $x_1 = \frac{1}{6}\pi + k\pi$, $x_2 = \frac{5}{6}\pi + k\pi$, $x_3 = \frac{7}{6}\pi + k\pi$, $x_4 = \frac{11}{6}\pi + k\pi$

6) Řešte v **R**:

$$\sin^2 x = \frac{1}{2}$$

VH: $(y + \frac{\sqrt{2}}{2})(y - \frac{\sqrt{2}}{2}) = 0$; $x_1 = \frac{1}{4}\pi + k\frac{\pi}{2}$

3. $\operatorname{tg} x \leftrightarrow \cot g x$

1) Řešte v **R**:

$$\operatorname{tg} x - 3 \cot g x = 0$$

Sb-MM: $(y + \sqrt{3})(y - \sqrt{3}) = 0$; $x_1 = \frac{1}{3}\pi + k\pi$, $x_2 = \frac{2}{3}\pi + k\pi \dots \text{str.77/5.2-c)}$

2) Řešte v **R**:

$$\operatorname{tg} x - 2 + \cot g x = 0$$

VH: $(y - 1)(y - 1) = 0$; $x_1 = \frac{1}{4}\pi + k\pi$

3) Řešte v **R**:

$$\cot g x - 2 + \operatorname{tg} x = 0$$

VH: $(y - 1)(y - 1) = 0$; $x_1 = \frac{1}{4}\pi + k\pi$

4) Řešte v **R**:

$$\operatorname{tg} x - \cot g x - \frac{2}{\sqrt{3}} = 0$$

Sb-MM: $(y - \sqrt{3})(y + \frac{1}{\sqrt{3}}) = 0$; $x_1 = \frac{1}{3}\pi + k\pi$, $x_2 = \frac{5}{6}\pi + k\pi \dots \text{str.77/5.2-d)}$

5) Řešte v **R**:

$$\operatorname{tg} x + \cot g x - \frac{4}{\sqrt{3}} = 0$$

VH: $(y - \sqrt{3})(y - \frac{1}{\sqrt{3}}) = 0$; $x_1 = \frac{1}{6}\pi + k\pi$, $x_2 = \frac{1}{3}\pi + k\pi$

6) Řešte v **R**:

$$\operatorname{tg} x - \cot g x + \frac{2}{\sqrt{3}} = 0$$

VH: $(y + \sqrt{3})(y - \frac{1}{\sqrt{3}}) = 0$; $x_1 = \frac{1}{6}\pi + k\pi$, $x_2 = \frac{2}{3}\pi + k\pi$

4. Základní kvadratická rovnice

1) Řešte v **R**:

$$2\sin^2 x - \sin x = 0$$

Sb-rce, VSP: $y(y - \frac{1}{2}) = 0$; $x_1 = \frac{1}{6}\pi + 2k\pi$, $x_2 = \frac{5}{6}\pi + 2k\pi$, $x_3 = k\pi \dots \text{str.174/5.4-1)}$

2) Řešte v **R**:

$$2\cos^2 x + 3\cos x + 1 = 0$$

Sb-rce: $(y + 1)(y + \frac{1}{2}) = 0$; $x_1 = \frac{2}{3}\pi + 2k\pi$, $x_2 = \frac{4}{3}\pi + 2k\pi$, $x_3 = \pi + 2k\pi \dots \text{str.174/5.4-4)}$

3) Řešte v **R**:

$$2\sin^2 x - \sin x - 1 = 0$$

VH: $(y - 1)(y + \frac{1}{2}) = 0$; $x_1 = \frac{7}{6}\pi + 2k\pi$, $x_2 = \frac{11}{6}\pi + 2k\pi$, $x_3 = \frac{\pi}{2} + 2k\pi$

4) Řešte v **R**:

$$4\sin^2 x - 4\sin x + 1 = 0$$

VH: $(y - \frac{1}{2})(y - \frac{1}{2}) = 0$; $x_1 = \frac{1}{6}\pi + 2k\pi$, $x_2 = \frac{5}{6}\pi + 2k\pi$

5) Řešte v **R**:

$$\sin^2 x + 2\sin x - 3 = 0$$

Sb-rce VSP: $(y - 1)(y + 3) = 0$; $x_1 = \frac{1}{2}\pi + 2k\pi$, $x_2 = N\check{R} \dots \text{str.174/5.4-2)}$

6) Řešte v **R**:

$$\sin^2 x + 5\sin x + 4 = 0$$

Sb-rce: $(y + 1)(y + 4) = 0$; $x_1 = \frac{3}{2}\pi + 2k\pi$, $x_2 = N\check{R} \dots \text{str.174/5.4-3)}$

7) Řešte v **R**:

$$\cos x(2\cos x + 1) = 1$$

Sb-rce, VSP: $(y + 1)(y - \frac{1}{2}) = 0$; $x_1 = \frac{1}{3}\pi + 2k\pi$, $x_2 = \frac{5}{3}\pi + 2k\pi$, $x_3 = \pi + 2k\pi \dots \text{str.174/5.4-5)}$

8) Řešte v **R**:

$$4\cos^2 x - 4\cos x - 3 = 0$$

$$\text{Sb-MM: } (y + \frac{1}{2})(y - \frac{3}{2}) = 0; \quad x_1 = \frac{2}{3}\pi + 2k\pi, x_2 = \frac{4}{3}\pi + 2k\pi, x_3 = N\check{R} \dots \text{str.77/5.1-e)}$$

9) Řešte v **R**:

$$\cot g^2 x = \sqrt{3} \cot gx$$

$$\text{Sb-MM: } y(y - \sqrt{3}) = 0; \quad x_1 = \frac{1}{2}\pi + k\pi, x_2 = \frac{1}{6}\pi + k\pi \dots \text{str.77/5.1-c)}$$

10) Řešte v **R**:

$$\sin x + \frac{1}{\sin x} = 2$$

$$\text{Sb-MM: } (y-1)(y-1) = 0; \quad x_1 = \frac{1}{2}\pi + 2k\pi \dots \text{str.77/5.1-g)}$$

11) Řešte v **R**:

$$\sin^2 x - 3\sin x - 10 = 0$$

$$\text{VSP: } (y-2)(y+5) = 0; \quad x_1 = N\check{R}, x_2 = N\check{R}$$

12) Řešte v **R**:

$$\sqrt{3}tg^2 x + 2tgx - \sqrt{3} = 0$$

$$\text{Sb-rce: } (y - \sqrt{3})(y + \frac{1}{\sqrt{3}}) = 0; \quad x_1 = \frac{1}{6}\pi + k\pi, x_2 = \frac{2}{3}\pi + k\pi \dots \text{str.174/5.4-7)}$$

13) Řešte v **R**:

$$\cot g^3 x = \cot gx$$

$$\text{Sb-rce: } y(y+1)(y-1) = 0; \quad x_1 = \frac{1}{2}\pi + k\pi, x_2 = \frac{1}{4}\pi + k\pi, x_3 = \frac{3}{4}\pi + k\pi \dots \text{str.174/5.4-8)}$$

5. $\sin^2 x \leftrightarrow \cos^2 x$

1) Řešte v **R**:

$$\sin x + 2\cos^2 x + \sin^2 x = 0$$

$$\text{VH: } (y+1)(y-2) = 0; \quad x_1 = \frac{3}{2}\pi + 2k\pi, x_2 = N\check{R}$$

2) Řešte v **R**:

$$\sin^2 x + \cos x + 1 = 0$$

$$\text{Sb-rce: } (y+1)(y-2) = 0; \quad x_1 = \pi + 2k\pi, x_2 = N\check{R} \dots \text{str.175/5.5-4)}$$

3) Řešte v **R**:

$$3\sin x + 2\cos^2 x + 3\sin^2 x = 0$$

$$\text{VH: } (y+1)(y+2) = 0; \quad x_1 = \frac{3}{2}\pi + 2k\pi, x_2 = N\check{R}$$

4) Řešte v **R**:

$$2\sin x - 3\cos^2 x - 2\sin^2 x = 0$$

$$\text{VH: } (y-1)(y+3) = 0; \quad x_1 = \frac{1}{2}\pi + 2k\pi, x_2 = N\check{R}$$

5) Řešte v **R**:

$$\frac{\cos x}{1 - \sin x} = 1 + \sin x$$

$$\text{Radl: } y(y-1) = 0; \quad x_1 = \frac{1}{2}\pi + k\pi, x_2 = 2k\pi$$

6) Řešte v **R**:

$$\frac{\sin x}{1 + \cos x} = 1 - \cos x$$

$$\text{Radl: } y(y-1) = 0; \quad x_1 = \frac{1}{2}\pi + 2k\pi, x_2 = k\pi$$

7) Řešte v **R**:

$$4\sin^2 x - 4\cos x - 1 = 0$$

$$\text{Radl: } (y - \frac{1}{2})(y - 3) = 0; \quad x_1 = \frac{1}{3}\pi + 2k\pi, x_2 = \frac{5}{3}\pi + 2k\pi, x_3 = N\check{R}$$

8) Řešte v **R**:

$$3\sin^2 x + \cos x + \cos^2 x = 0$$

$$\text{Sb-MM: } (y+1)(y - \frac{3}{2}) = 0; \quad x_1 = \pi + 2k\pi, x_2 = N\check{R} \dots \text{str.77/5.2-b)}$$

9) Řešte v **R**:

$$2\sin^2 x + 7\cos x - 5 = 0$$

$$\text{G-Pe: } (y - \frac{1}{2})(y - 3) = 0; \quad x_1 = \frac{1}{3}\pi + 2k\pi, x_2 = \frac{5}{3}\pi + 2k\pi, x_3 = N\check{R}$$

10) Řešte v **R**:

$$6\sin^2 x - 7\cos x - 1 = 0$$

$$\text{Sb-rce: } (y - \frac{1}{2})(y + \frac{5}{3}) = 0; \quad x_1 = \frac{1}{3}\pi + 2k\pi, x_2 = \frac{5}{3}\pi + 2k\pi, x_3 = N\check{R} \dots \text{str.175/5.5-1)}$$

11) Řešte v **R**:

$$3\sin x = 2\cos^2 x$$

$$\text{Sb-rce: } (y - \frac{1}{2})(y + 2) = 0; \quad x_1 = \frac{1}{6}\pi + 2k\pi, x_2 = \frac{5}{6}\pi + 2k\pi, x_3 = N\check{R} \dots \text{str.175/5.5-2)}$$

12) Řešte v **R**:

$$\sin x + \cos^2 x = \frac{1}{4}$$

$$\text{Sb-rce: } (y + \frac{1}{2})(y - \frac{3}{2}) = 0; \quad x_1 = \frac{7}{6}\pi + 2k\pi, x_2 = \frac{11}{6}\pi + 2k\pi, x_3 = N\check{R} \dots \text{str.175/5.5-5)}$$

13) Řešte v **R**:

$$3\cos x = 2\sin^2 x$$

$$\text{Sb-rce: } (y - \frac{1}{2})(y + 2) = 0; \quad x_1 = \frac{1}{3}\pi + 2k\pi, x_2 = \frac{5}{3}\pi + 2k\pi, x_3 = N\check{R} \dots \text{str.175/5.5-6)}$$

14) Řešte v **R**:

$$2\sqrt{3}\sin^2 x = \cos x$$

$$\text{Sb-rce: } (y + \frac{\sqrt{3}}{2})(y - \frac{2}{\sqrt{3}}) = 0; \quad x_1 = \frac{5}{6}\pi + 2k\pi, x_2 = \frac{7}{6}\pi + 2k\pi, x_3 = N\check{R} \dots \text{str.175/5.5-3)}$$

15) Řešte v **R**:

$$3\sin^2 x = \cos^2 x$$

$$\text{Sb-MM: } \dots \text{str.77/5.2-a)}$$

$$(y - \frac{1}{2})(y + \frac{1}{2}) = 0; \quad x_1 = \frac{1}{6}\pi + 2k\pi, x_2 = \frac{5}{6}\pi + 2k\pi, x_3 = \frac{7}{6}\pi + 2k\pi, x_4 = \frac{11}{6}\pi + 2k\pi$$

16) Řešte v **R**:

$$\cos^2 x + \cos x = \sin^2 x$$

$$\text{VH: } (y - \frac{1}{2})(y + 1) = 0; \quad x_1 = \frac{1}{3}\pi + 2k\pi, x_2 = \frac{5}{3}\pi + 2k\pi, x_3 = \pi + 2k\pi$$

17) Řešte v **R**:

$$\cos^2 x + \sin x - \frac{1}{4} = 0$$

$$\text{Radl: } (y + \frac{1}{2})(y - \frac{3}{2}) = 0; \quad x_1 = \frac{7}{6}\pi + 2k\pi, x_2 = \frac{11}{6}\pi + 2k\pi, x_3 = N\check{R}$$

18) Řešte v **R**:

$$\sqrt{2}\sin x + \cot gx = 0$$

$$\text{Radl: } (y + \frac{\sqrt{2}}{2})(y - \sqrt{2}) = 0; \quad x_1 = \frac{3}{4}\pi + 2k\pi, x_2 = \frac{5}{4}\pi + 2k\pi, x_3 = N\check{R}$$

19) Řešte v **R**:

$$\cos x - \frac{1}{\sqrt{2}}tgx = 0$$

$$\text{Radl: } (y - \frac{\sqrt{2}}{2})(y + \sqrt{2}) = 0; \quad x_1 = \frac{1}{4}\pi + 2k\pi, x_2 = \frac{3}{4}\pi + 2k\pi, x_3 = N\check{R}$$

20) Řešte v **R**:

$$\frac{\sin x(1 + \sin x - \cos^2 x)}{\cos^2 x(1 + \sin x)} = \frac{1}{3}$$

Radl: $(y + \frac{1}{2})(y - \frac{1}{2}) = 0$; $x_1 = \frac{1}{6}\pi + k\pi, x_2 = \frac{5}{6}\pi + k\pi$

21) Řešte v **R**:

$$\frac{\cos x(1 + \cos x - \sin^2 x)}{\sin^2 x(1 + \cos x)} = 3$$

Radl: $(y + \frac{\sqrt{3}}{2})(y - \frac{\sqrt{3}}{2}) = 0$; $x_1 = \frac{1}{6}\pi + k\pi, x_2 = \frac{5}{6}\pi + k\pi$